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Exotic prepotentials from D(-1)D7 dynamics

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Abstract: We compute the partition functions of D(-1)D7 systems describing the multi-instanton dynamics of SO(N) gauge theories in eight dimensions. This is the simplest instance of the so called exotic instantons. In analogy with the Seiberg-Witten theory in four space-time dimensions, the prepotential and correlators in the chiral ring are derived via localization formulas and found to satisfy relations of the Matone type. Exotic prepotentials of SO(N) gauge theories with $\mathcal{N} = 2$ supersymmetries in four-dimensions are also discussed.

Keywords: Brane Dynamics in Gauge Theories, Intersecting branes models, Nonperturbative Effects, Topological Field Theories

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Contents

1 Introduction

Extracting the standard model or some supersymmetric (SUSY) extension of it from a D-brane construction has been the focus of much recent work (see [1–3] for recent reviews). In this framework, it has been recently understood that certain non perturbative effects may lead to interesting new phenomena in low energy theories like a generation of a Majorana mass term or of Yukawa couplings [4, 5], not to mention new possible patterns of SUSY breaking [6]–[14]. This proposal was further sharpened by the observation that to really have such effects, orientifold planes [15]–[20] or closed string fluxes [21, 22] need to be introduced.

To the extent of understanding non-perturbative effects in systems of D-branes a derivation of four dimensional instantons in terms of D-branes was carried out in [23–25]. These results where re-discussed in [26] where the field theory results were obtained by string methods and a careful parallel between the ADHM formalism and the system formed by bound states of parallel D(-1) and D3 branes was carried out. The reader should be aware that the non perturbative effects alluded to in the previous paragraph are not of this type. In general bound states of intersecting branes at angles (with more than four Neuman-Dirichlet directions) or branes hosting generic non parallel magnetic fluxes lack those bosonic moduli related to the instanton sizes and gauge orientations. This type of instantons have been called exotic and besides their phenomenological applications are interesting in itself since they are relevant for many non perturbative effects in string theory.

We will then focus our attention on the D(-1)D7 system in the presence of an orientifold O7 plane. This is the simplest supersymmetric instance of an exotic instanton. We choose

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the orientifold projection in such a way to get an SO(N) gauge theory with SO(k) exotic instantons which carry the right number of zero modes to generate a potential [16, 17]. From the low energy gauge theory viewpoint, this is an eight dimensional instanton. As it is well-known, extending the idea of self-duality to dimensions greater than four is far from obvious. Self-duality in four dimensions is tantamount to say that the field strength is a $(1,1)$ complex form of Einstein-Kähler type. This was the starting point of the first explorations in this field [27]. Later a solution to the quadratic Yang-Mills (YM) action in eight dimensions was found [28] with gauge group SO(7). Finally an SO(8) gauge connection was exhibited [29, 30] which was the generalization of the Hopf map $S^7 \xrightarrow{S^3} S^4$ in four dimensions to eight dimensions where the SO(8) gauge bundle is thought as the Hopf map $S^{15} \xrightarrow{S^7} S^8$. This latter solution has been recently related to D(-1) instantons on the SO(8) gauge theory living in a D7 O7 worldvolume [31]: this interpretation is supported by the observation [31] that in the limit of instanton zero size the quadratic YM term vanishes. Moreover the quartic term, F^4, becomes proportional to the fourth Chern class of the gauge bundle thus matching the D(-1) action. This proposal have been put on solid grounds in [32] where instanton corrections to F^4-terms were computed in complete analogy with the four dimensional case [33], [39] finding agreement with the heterotic results [40], [47]. We remark that an ADHM construction of the moduli space of these "exotic" eight dimensional instantons is missing and therefore these D-brane techniques are at present the only way to investigate this physics.

The SO(8) gauge theory in eight dimensions is very special. The theory is conformal, in the sense that the coupling τ_4 of the F^4-term does not run. Conformal invariance implies that τ_4 is constant over the moduli space since it cannot depend on dimensionful quantities such as the vevs of the scalar field. Unlike in the conformal case, in the case of SO(N), the coupling τ_4 runs logarithmically with a one loop beta coefficient $\beta_4 = \frac{1}{2}(8-N)$. In this paper we will deal with the non-conformal case: our gauge group will be SO(N) with $N > 8$ and, therefore, $\beta_4 < 0$. Like for Seiberg-Witten theory in four-dimensions, in the non-conformal case the instanton measure acquires a dimension and the F^4 coupling becomes a non trivial function of the scale generated by the exotic instantons and the casimirs parametrizing the moduli space. The aim of this paper is to address the study of multi-instanton corrections to the effective action of such eight-dimensional gauge theories. We will also consider the case of $\mathcal{N} = 2$ gauge theories in four dimensions which arises from placing the D(-1)D7 system at a $\mathbb{R}^4/\mathbb{Z}_2$ singularity that freezes the dynamics along the orbifold four-plane. How to test these results against heterotic computations with non trivial Wilson lines remains an open and challenging task.[1]

This is the plan of the paper: in section 2 and section 3 we review the main features of the eight dimensional instanton and the localization algorithm. In section 4 and section 5 we compute the exotic prepotentials describing the dynamics of SO(N) gauge theories in $d = 8$ and $d = 4$ dimensions respectively. Finally in section 6 we compute the correlators of the chiral ring.

[1]We thank M. Bianchi for discussions on this issue.

2 Eight dimensional instantons

In this section we review the proposal in [31, 32] for a D(-1)D7 description of the SO(8) eight-dimensional instantons and extend it to the non conformal case with gauge group SO(N). The field content of maximal supersymmetric YM theory in eight dimensions includes a gauge boson with field strength F, a complex scalar ϕ and two fermions of opposite chirality. In complete analogy with the $\mathcal{N}=2$ SYM in $d=4$ dimensions the chiral dynamics of the SYM theory in eight dimensions is described by a prepotential $\mathcal{F}(\Phi)$ in terms of which the effective action can be written as

$$S_{\rm eff} = \int d^8x d^8\theta\, \mathcal{F}(\Phi) + \text{h.c.} \tag{2.1}$$

with

$$\Phi = \phi + \sqrt{2}\theta\Psi + F_{\mu\nu}\theta\gamma^{\mu\nu}\theta + \dots \tag{2.2}$$

the chiral eight-dimensional superfield. At the classical level $\mathcal{F}_{\rm cl} = i\tau_4\,{\rm Tr}\,\Phi^4$ and the effective action becomes

$$S_{\rm eff} = {\rm Re}\,\tau_4 \int_{\mathbb{R}^8} d^8x\, t_8\, F^4 + i{\rm Im}\,\tau_4 \int_{\mathbb{R}^8} F\wedge F\wedge F\wedge F \tag{2.3}$$

with

$$\tau_4 = \frac{\theta}{2\pi} + i\frac{4!(2\pi)^3}{g^2} \tag{2.4}$$

and where t_8 is the invariant eight-rank tensor of SO(8). As we said in the introduction, D(-1) instantons are identified with zero size instantons with vanishing quadratic YM action and therefore we will always discard this term from the effective action. The gauge theory can be realized in terms of a stack of N D7-branes on top of a O7-plane. Instantons correct the prepotential function $\mathcal{F}(\Phi)$ and then the effective action. Instantons in the eight-dimensional gauge theory are realized in terms of D(-1) branes with open strings describing the instanton moduli space and the D(-1)D7 action gives the gauge dynamics around the instanton background. By instantons here we refer to solutions of the Yang-Mills equations with action

$$S_{\rm cl} = 2\pi\,\tau_4\, k \tag{2.5}$$

with k the fourth Chern class

$$k = \frac{1}{4!(2\pi)^4}\int_{\mathbb{R}^8} {\rm Tr}\, F\wedge F\wedge F\wedge F \tag{2.6}$$

An explicit solution in this class can be written as[2]

$$F_{\mu\nu} = -\frac{2\rho^2}{(x^2+\rho^2)^2}\,\gamma_{\mu\nu} \tag{2.7}$$

[2] Using the properties of SO(8) gamma functions one finds that F satisfy $*(F\wedge F) = F\wedge F$ and $*F = T\wedge F$ with $T_{\mu\nu\sigma\rho} = \eta\,\gamma_{[\mu}\,\gamma^\dagger_\nu\,\gamma_\sigma\,\gamma^\dagger_{\rho]}\,\eta$ with η a fixed eight dimensional spinor. These two relations define possible generalizations of the concept of four-dimensional self-duality to eight dimensions.

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with

$$\gamma_{\mu\nu} = \Upsilon \begin{pmatrix} 0 & 0 \\ 0 & \gamma_{\mu\nu}^{\mathrm{SO}(8)} \end{pmatrix} \Upsilon^T \tag{2.8}$$

and $\Upsilon \in \mathrm{SO}(N)/\mathrm{SO}(N-8)$ parametrizing the orientation of the SO(8) instanton inside SO(N). $\gamma_{\mu\nu}^{\mathrm{SO}(8)} = \gamma_{[\mu}\gamma_{\nu]}^{\dagger}$ with γ_μ the SO(8) gamma matrices satisfying $\gamma_{(\mu}\gamma_{\nu)}^{\dagger} = \delta_{\mu\nu}$. This is the solution originally found in [29, 30]. In the limit $\alpha', \rho \to 0$ the quadratic YM action evaluated at this solution vanishes and the quartic term matches that of the D(-1) instanton with $\theta = C_0$ the RR 0-form and $g^2 = g_s$ the string coupling [31].

The study of the D(-1)D7 dynamics follows closely that of its four-dimensional D(-1)D3 analog with some important differences. First the O7-orientifold projection acts with the same sign on the D7 and D(-1) Chan-Paton indices. This implies in particular that the symmetry group of k D(-1) instantons in the SO(N) gauge theory coming from the D7-branes is SO(k). This is in contrast with the four-dimensional case where SO(N) instantons carry an Sp(k) symmetry group. Secondly, unlike in the D(-1)D3 case, open strings between D(-1) and D7 branes have no bosonic zero modes and therefore interactions between the two brane stacks are mediated only via the fermionic field ν (the Ramond ground state) in the bifundamental of the SO(N) $\times$ SO(k) symmetry group. Like in the four-dimensional case, the correlators in the gauge theory can be written as the moduli space integral [17, 32]

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \sum_k e^{2\pi i k \tau_4(\mu)} \, \mu^{k\beta_4} \int d\mathfrak{M}_{k,N} \, e^{-S_k - \nu^T \phi \nu} \, \mathcal{O} \tag{2.9}$$

with $\tau_4(\mu)$ the logarithmically running F^4-coupling, β_4 its one-loop beta function coefficient and μ the cut-off energy scale regularizing the one-loop vacuum integrals (Annulus and Moebius amplitudes with an end on the D7-branes) . Like in QCD, this regulator defines the "renormalization group invariant" $q = \Lambda^{\beta_4} = \mu^{\beta_4} e^{2\pi i \tau_4(\mu)}$ with no dependence on μ. S_k is the instanton action following from the dimensional reduction of $\mathcal{N} = 1$ $d = 10$ SYM down to zero dimensions and $\mathfrak{M}_{k,N}$ the D(-1)D7 moduli space. Finally $Z = \langle \mathbb{I} \rangle$ is the instanton partition function. Later we will also consider correlators of the form $\langle \mathrm{Tr}\phi^J \rangle$. In particular the basic correlator $\langle \mathrm{Tr}\phi^4 \rangle$ will be related to the prepotential of the eight dimensional theory.

We remark that Φ couples to the D(-1) instantons only through the Yukawa coupling $\nu^T \phi \nu$. Integration over ν leads to a polynomial dependence on ϕ. This in particular allows for generation of phenomenologically interesting Majorana mass terms and Yukawa couplings in the effective action. This is in sharp contrast with the standard gauge instanton potentials which fall off to zero in the limit of large vev of the scalar field. Another important difference with the four-dimensional case is that the weak coupling regime $q \to 0$ corresponds to the large energy limit $\mu \to \infty$. This can be seen from the relation $q = \Lambda^{\beta_4}$ and the fact that $\beta_4 = \frac{1}{2}(8 - N) < 0$, as already pointed out in the introduction.

3 Localization formulae

In this section we review the localization algorithm developed in [33–36] to study integrals over the instanton moduli space. The main idea is the identification of an equivariantly

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deformed BRST operator Q_ξ, satisfying $Q_\xi^2 = \mathcal{L}_\xi$ with $\mathcal{L}_\xi$ a Lie derivative on the moduli space along a vector field ξ belonging to the Cartan of the $\mathrm{SO}(K) \times \mathrm{SO}(N) \times \mathrm{SO}(8)$ symmetry group. ξ can be parameterized by $\xi = (\chi_i, a_u, \epsilon_\ell)$ with $i = 1,..k$, $u = 1,..n$, $\ell = 1,..4$ ($n = [N/2], k = [K/2]$). Choosing a generic ξ the symmetry group is broken to its Cartan part and the integral is given by the contributions at isolated critical points (the poles in a contour integral over χ_i). Physical quantities in the gauge theory are defined by taking the limit $\epsilon_\ell \to 0$ in order to eliminate the singularity arising from the D(-1) branes all superposed at the origin. More precisely, the prepotential $F(\Phi)$ is given by the formula

$$F(a_u) = \lim_{\epsilon_\ell \to 0} \left[\epsilon_1 \epsilon_2 \epsilon_3 \epsilon_4 \ln \sum_k Z_k(a_u, \epsilon_\ell) q^k \right] \tag{3.1}$$

with Z_k the instanton partition function. Keeping ϵ_ℓ finite one finds the gravitational corrections to the Yang-Mills action [49]. The instanton partition function $Z_k(a_u, \epsilon_\ell)$ can be written as an integral over the instanton moduli space. The integral can be performed using the localization formula[3]

$$Z_k = \int d\mathfrak{M}_{k,N} e^{-S_{\text{inst}}} = \int \frac{d^k \chi_i}{\mathrm{Sdet} Q_\xi^2} = \int d^k \chi_i \prod_\Phi \lambda_\Phi(\chi, a, \epsilon)^{(-)^{F_\Phi}+1} \tag{3.2}$$

where Φ labels the Q-multiplet pairs (Φ, Ψ) related by the BRST transformations

$$Q\,\Phi = \Psi \qquad Q\Psi = \lambda_\Phi(\chi, a, \epsilon)\,\Phi \tag{3.3}$$

$F_\Phi = 0\,(1)$ for Φ a bosonic (fermionic) field and λ_Φ is the eigenvalues of Q_ξ^2. It is important to notice that using $Q \sim \mu^{\frac{1}{2}}$ the dimension of the moduli space measure can be written as

$$d\mathfrak{M}_{k,N} \sim \mu^{\frac{1}{2}(n_F - n_B)} \tag{3.4}$$

with n_B, n_F the number of Q-multiplets with lowest component a boson and a fermion respectively.

The topological theory for the system discussed in the previous section, describes the excitations of open strings connecting the various branes with at least one end on the D(-1) instanton. We denote the fields by $\Phi_{\mathcal{AB}}$ where the index $\mathcal{A} = (I, U)$ runs over all possible boundaries $I = 1, \ldots .K$ (number of D(-1) branes), $U = 1, \ldots N$ (number of D7-branes). In presence of an O7 plane, let us say at $x = 0$ in the transverse plane, the branes are distributed symmetrically with respect to it. We denote by $x_{\mathcal{A}} = (\chi_I, a_U)$ the positions of the various branes

$$\begin{aligned} \chi_I &= (\chi_1, \ldots ., \chi_k; -\chi_1, \ldots, -\chi_k; 0) \\ a_U &= (a_1, \ldots .a_n, -a_1, \ldots ., -a_n; 0) \end{aligned} \tag{3.5}$$

The last "0" should be omitted in the case of an even number of branes. The BRST operator is defined by equivariantly deforming the SUSY algebra by the $\mathrm{SO}(K) \times \mathrm{SO}(N)$

[3]The integral over the moduli space follows from the eight dimensional gauge theory after having performed a quadratic saddle point approximation and having substituted the fields with their classical solution around the saddle. The localization formula is then an exact evaluation of such integral.

brane symmetry. In addition complete localization requires also a $\Upsilon(1)^3$ deformation inside the Lorentz SO(8) group parametrized by ϵ_ℓ, $\ell = 1, \dots 4$ with $\sum_\ell \epsilon_\ell = 0$. More precisely the Q^2-eigenvalue of a field $\Phi_{\mathcal{AB}}$ can be written as

$$\lambda_\Phi = x_\mathcal{A} - x_\mathcal{B} + q_\Phi \tag{3.6}$$

with q_Φ the $\Upsilon(1)^3$ charge of the given field. Taking into account that the presence of the orientifold halves the degrees of freedom in the covering space the partition function can then be written as

$$Z_K = \int d^k\chi_i \prod_{\mathcal{A},\mathcal{B},\Phi}{}' (x_\mathcal{A} - x_\mathcal{B} + q_\Phi)^{\frac{1}{2}(-)^{F_\Phi+1}} \prod_{\mathcal{A},\Phi} (2\, x_\mathcal{A} + q_\Phi)^{\frac{1}{2}(-)^{F_\Phi+1}\delta_\Phi} \tag{3.7}$$

with $\delta_\Phi = \pm$ depending on whether the field is even or odd under the orientifold projection. The primed product runs over all $\mathcal{A}, \mathcal{B}$ pairs with at least one index on the D(-1) instantons. The second contribution comes from open strings connecting the D-brane $\mathcal{A}$ to its image. It is important to notice that despite the explicit appearance of square roots, from (3.5) one can see that each eigenvalue appears twice and therefore the final answer contains no square roots.

The BRST transformations for the various strings under consideration are:

- D(-1)D(-1) open strings

$$\begin{aligned} Q\, B_{\ell;IJ} &= M_{\ell;IJ} \qquad Q\, M_{\ell;IJ} = (\chi_{IJ} + \epsilon_\ell)\, B_{\ell;IJ} \\ Q\, \lambda_{c;IJ} &= D_{c;IJ} \qquad Q\, D_{c;IJ} = (\chi_{IJ} + s_c)\, \lambda_{c;IJ} \end{aligned} \tag{3.8}$$

with $\ell = 1, ..4,\ ,\ c = 1, ..4$ and

$$\begin{aligned} s_1 = \epsilon_2 + \epsilon_3 \qquad s_2 = \epsilon_1 + \epsilon_3 \qquad s_3 = \epsilon_1 + \epsilon_2 \qquad s_4 = 0 \\ \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 = 0 \end{aligned} \tag{3.9}$$

In writing the BRST multiplets we group the seven ADHM constraints D_s, $s = 1, \dots 7$ and the component $\bar\chi$ into four complex fields denoted by D_c. In doing this we should remind that the zero eigenvalues associated to the diagonal field components $(\lambda_4, D_4)_{II}$ should be omitted from the determinant. Alternatively the contribution of this pair to the partition function can be thought as coming from the Vandermonde determinant resulting from bringing the field χ into its diagonal form.

The orientifold projection project a_ℓ and λ_c on symmetric and antisymmetric matrices respectively i.e.

$$\delta_a = + \qquad \delta_\lambda = - \tag{3.10}$$

This is consistent with the fact that the ADHM contraints D_c can be written in terms of commutators of a_ℓ.

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(Φ,Ψ)	$(-)^{F_\Phi}$	δ_Φ	multiplicity	q_Φ
(B_ℓ, M_ℓ)	$+$	$+$	$8\,\frac{1}{2}\,K(K+1)$	ϵ_ℓ
(λ_c, D_c)	$-$	$-$	$8\,\frac{1}{2}\,K(K-1)$	s_c
$(\,\nu, h)$	$-$	0	$2nK$	0

Table 1. D(-1)D7 open strings moduli.

- D(-1)D7 open strings

$$Q\,\nu_{IU} = h_{IU} \qquad Q\,h_{IU} = (\chi_I - a_U)\,\nu_{IU} \tag{3.11}$$

The field h is an auxiliary field needed to close the Q-algebra [50].

4 D(-1)D7 on $\mathbb{R}^{10}$

In this section we consider a system of K D(-1) branes, $N = 2n$ D7-branes and an O7-plane realizing a maximal SUSY SO($2n$) gauge theory in eight dimensions. The total symmetry group is then SO($2n$) $\times$ SO(K). Fields like the fermionic ADHM auxiliary fields λ_c transforming in the adjoint of SO(K) group are described by antisymmetric matrices while instanton positions B_ℓ are given in terms of symmetric matrices. The field content in the instanton moduli space is summarized in table 1. Plugging these data into the general formula (3.7) one finds the partition function

$$Z_K = \mathcal{N}_K \int \prod_{i=1}^{k} \frac{d\chi_i}{2\pi i} \prod_{I,J}^{K} \left[\frac{P(\chi_{IJ})}{Q(\chi_{IJ})}\right]^{\frac{1}{2}} \prod_{I=1}^{K} \left[\frac{M(\chi_I)}{P(2\chi_I)Q(2\chi_I)}\right]^{\frac{1}{2}} \tag{4.1}$$

with $\chi_{IJ} = \chi_I - \chi_J$ and

$$\begin{aligned} P(x) &= x^{1-\delta_{x,0}} \prod_{a=1}^{3} (x + s_a), \\ Q(x) &= \prod_{\ell=1}^{4} (x + \epsilon_\ell) \\ M(x) &= \prod_{u=1}^{n} (x + a_u) \end{aligned} \tag{4.2}$$

These polynomials give the contribution of the fields λ_c, B_ℓ and ν respectively. Notice that $(-)^{F_\Phi} = \delta_\Phi$ for $\Phi = \lambda_c, B_\ell$ explaining why both contributions $P(2\chi_I), Q(2\chi_I)$ come in the denominator.

Setting $K = 2k$ and $K = 2k+1$ and using (3.5) one finds

$$
\begin{aligned}
Z_{2k} &= \mathcal{N}_{2k} \int \prod_{i=1}^{k} \frac{d\chi_i}{2\pi i} \prod_{i\le j}^{k} \frac{P_2(\chi_{ij}^-)P_2(\chi_{ij}^+)}{Q_2(\chi_{ij}^-)Q_2(\chi_{ij}^+)} \prod_{i=1}^{k} \frac{M_2(\chi_i)}{P_2(2\chi_i)} \qquad (4.3)\\
Z_{2k+1} &= \mathcal{N}_{2k+1} \frac{M(0)}{Q_2(0)} \int \prod_{i=1}^{k} \frac{d\chi_i}{2\pi i} \prod_{i\le j}^{k} \frac{P_2(\chi_{ij}^-)P_2(\chi_{ij}^+)}{Q_2(\chi_{ij}^-)Q_2(\chi_{ij}^+)} \prod_{i=1}^{k} \frac{M_2(\chi_i)P_2(\chi_i)}{P_2(2\chi_i)Q_2(\chi_i)}
\end{aligned}
$$

with

$$
\begin{aligned}
P_2(x) &= (-x^2)^{1-\delta_{x,0}} \prod_{a=1}^{3} (s_a^2 - x^2), \\
Q_2(x) &= \prod_{\ell=1}^{4} (\epsilon_\ell^2 - x^2)\,, \\
M_2(x) &= \prod_{u=1}^{n} (a_u^2 - x^2)\,, \\
\mathcal{N}_{2k} &= \frac{2^{4(2k)}}{2^k k!}\ ; \quad \mathcal{N}_{2k+1} = \frac{2^{4(2k+1)}}{2^k k!}\,. \qquad (4.4)
\end{aligned}
$$

Integrals over χ_i should be supplemented with a pole prescription. Following the four dimensional analogy we take $\epsilon_\ell \to \epsilon_\ell + i\delta_\ell$ with $\delta_1 \gg \delta_2 \gg \delta_3 \gg \delta_4$. The prepotential of the eight dimensional theory can be extracted from the relation

$$
F(a_u, \epsilon_\ell) = \epsilon_1\epsilon_2\epsilon_3\epsilon_4 \ln \sum_K Z_K(a_u, \epsilon_\ell) q^K \qquad (4.5)
$$

An explicit evaluation of the integrals for the first few instanton contributions leads to

$$
\begin{aligned}
F(a,\epsilon) &= 8\sqrt{A_n}\left(q + \frac{4}{3}\, q^3\, A_{n-4} - \frac{5}{6}\, q^3\, A_{n-5} g_2 + \frac{1}{96}\, q^3\, A_{n-6}\left(25 g_2^2 + 34 g_4\right) + \ldots\right) \\
&\quad + q^2 \left(-2A_{n-2} + \frac{1}{4}\, A_{n-3} g_2 - \frac{1}{64} A_{n-4}\left(g_2^2 + 2g_4\right) + \ldots\right) \qquad (4.6)
\end{aligned}
$$

with A_m, $m = 1, \ldots n$ the mth elementary symmetric functions of the variables $a_1^2, \ldots, a_n^2$:

$$
\begin{aligned}
A_s &= \sum_{i_1 < i_2 \ldots < i_s} a_{i_1}^2 \ldots a_{i_s}^2 \\
A_n &= a_1^2 \ldots a_n^2 \qquad A_0 = 1 \qquad A_{s<0} = 0 \qquad (4.7)
\end{aligned}
$$

Notice that A_m form a basis for the Casimirs of SO$(2n)$. Similarly we parametrize the SO(8) Casimirs in terms of

$$
g_{2m} = \sum_{\ell=1}^{4} \epsilon_\ell^{2m} \qquad (4.8)
$$

In the appendix we present also 4 and 5 instanton contributions.

The effective action follows by replacing in the prepotential F the lowest components a_u, ϵ_ℓ by the corresponding chiral and gravitational superfields

$$
\begin{aligned}
a_u &\to \Phi_u = \phi_u + F^u_{\mu\nu}\theta\gamma^{\mu\nu}\theta + \ldots \\
\epsilon_\ell &\to W_\ell = G_\ell + R^\ell_{\mu\nu}\theta\gamma^{\mu\nu}\theta + \ldots
\end{aligned} \tag{4.9}
$$

Here we denote by G_ℓ the graviphoton with $\ell = 1,..4$ and $u = 1,..n$ running over the Cartan subgroup components of the Lorentz SO(8) and gauge SO($2n$) symmetry groups respectively.

The prepotential $F(\Phi, W)$ encodes the chiral dynamics of the $d = 8$ gauge theory coupled to gravity. The eight-dimensional effective action follows from $F(\Phi, W)$ upon integration over the chiral superspace variables

$$
S_{\text{eff}} = \int d^8x d^8\theta \, F(\Phi, W) \tag{4.10}
$$

In absence of gravity $W = 0$, it gives a direct analog of the Seiberg-Witten prepotential for $\mathcal{N} = 2$ gauge theories in $d = 4$ dimensions. For instance the F^4 coupling

$$
S_{\text{eff}} = \tau_4(a) \int d^8x \, \mathrm{tr}\, t_8 \, F^4 \tag{4.11}
$$

can be expressed as the fourth derivative of the prepotential

$$
\tau_4(A) = \left[\sum_{u=1}^{n} \frac{\partial^4}{\partial a_n^4} - \frac{1}{12} \left(\sum_{u=1}^{n} \frac{\partial^2}{\partial a_n^2} \right)^2 \right] F(a, 0) \tag{4.12}
$$

Notice that using (3.4) the instanton measure goes like

$$
d\mathfrak{M} \sim \mu^{\frac{1}{2}(n_\lambda + n_\nu - n_a)} = \mu^{K(n-4)} \tag{4.13}
$$

This implies that a dimensionless partititon function $Z = \sum_K Z_K q^K$ can be defined taking $q = \mu^{4-n} e^{2\pi i \tau_4}$. The prepotential of the eight-dimensional theory defined in terms of $Z(q, A)$ will then carry a non-trivial dependence on the scale $q = \Lambda^{4-n}$ generated by the exotic instantons and on the Casimir invariants A_n's parametrizing the moduli space of the theory. The case $n = 4$ is special in the sense that q is dimensionless and the theory is conformal: this is the case that has been studied in details in [32]. We recover in appendix B their results.

5 D(-1)D7 on $\mathbb{R}^6 \times \mathbb{R}^4/\mathbb{Z}_2$

The analysis in the last section can be extended to theories with less supercharges and lower dimensions. Here we consider the $\mathcal{N} = 2$ case in four-dimensions. This theory can be realized by considering a set of $2n$ fractional D7-branes wrapping a $\mathbb{R}^4/\mathbb{Z}_2$-singularity. We choose the $\mathbb{Z}_2$ orbifold group to act trivially in the Chan Paton indices. This projects out the gauge components along the four directions acted by the $\mathbb{Z}_2$ leaving an effective four-dimensional theory with $\mathcal{N}=2$ supersymmetries. On the instanton moduli space the orbifold groups acts like

$$a_{3,4} \to -a_{3,4} \qquad M_{3,4} \to -M_{3,4} \qquad \lambda_{1,2} \to -\lambda_{1,2} \qquad D_{1,2} \to -D_{1,2} \tag{5.1}$$

with no action on the Chan-Paton indices and all the other fields invariant. In the language of fractional branes this corresponds to take $(n_0, n_1) = (2n, 0)$ D7-branes and $(k_0, k_1) = (K, 0)$ D(-1) branes.

The partition function follows now from the previous results by simply suppressing the contribution of the odd fields. The results are given again by (4.3) but with the characteristic functions replaced by

$$\begin{aligned} P_2(x) &= x^{2-2\delta_{x,0}}(x^2-\epsilon^2) \\ Q_2(x) &= (x^2-\epsilon_1^2)(x^2-\epsilon_2^2) \\ M_2(x) &= \prod_{u=1}^{n}(x^2-a_u^2) \end{aligned} \tag{5.2}$$

and $\epsilon = \epsilon_1 + \epsilon_2$.

Now the prepotential defines the four-dimensional effective action

$$S_{\text{eff}} = \int d^4x d^4\theta\, F(\Phi, W) = \tau_2(A) \int d^4x \,\mathrm{tr}\, F^2 + \ldots \tag{5.3}$$

where $\tau_2(A)$ is given by the second derivative of the prepotential. Notice that despite the similarities the instantons contributing to $\tau_2(A)$ are exotic and therefore the structure of $\tau_2(A)$ will be very different from that following from Seiberg-Witten type geometries. In particular, on the contrary of the prepotentials found in [51, 52], the Casimir A_m's appear in $\tau_2(A)$ only in a polynomial form.

The first terms in the expansion of the prepotential in the instanton winding number are given by

$$\begin{aligned} F(a,\epsilon) = 2\sqrt{A_n}\Bigg[& q + \frac{1}{3}q^3\left(A_{n-2} - A_{n-3}\left(\frac{7}{4}g_2 + \frac{9}{4}\epsilon_1\epsilon_2\right)\right. \\ & \left.\left. + A_{n-4}\left(\frac{13}{32}g_4 + \frac{49}{32}g_2^2 + \frac{45}{16}g_2\epsilon_1\epsilon_2\right)\right) + \cdots \right] \\ & + \frac{1}{2}q^2\left[A_{n-1} - A_{n-2}\left(\frac{1}{4}g_2 + \frac{1}{4}\epsilon_1\epsilon_2\right) + A_{n-3}\left(\frac{1}{16}g_4 + \frac{1}{16}g_2\epsilon_1\epsilon_2\right)\right] + \cdots\,, \end{aligned} \tag{5.4}$$

where $g_2 = \epsilon_1^2 + \epsilon_2^2$ and $g_4 = \epsilon_1^4 + \epsilon_2^4$, The case of SO(4) gauge group is of particular interest. In this case the theory is conformal and the instanton parameter q dimensionless. Calculation of up to q^7 terms suggests that the all orders exact prepotential is

$$F_{\mathrm{SO}(4)}(\phi, G) = \mathrm{Pf}\,\phi \log\frac{1+q}{1-q} + \left(\frac{1}{4}\,tr\,\phi^2 - \frac{1}{16}\,tr\,G^2 + \frac{1}{8}\,\mathrm{Pf}\,G\right)\log\left(1-q^2\right), \tag{5.5}$$

where

$$tr\,\phi^2 = -2(a_1^2 + a_2^2); \quad tr\,G^2 = -2g_2; \quad \mathrm{Pf}\,G = \epsilon_1\epsilon_2\,. \tag{5.6}$$

6 Chiral ring

The techniques developed in the previous sections apply as well to the computation of the general chiral correlator $tr\,\phi^J$ in the gauge theory. These correlators constitute the so called "chiral ring". In complete analogy with the 4 d $\mathcal{N}=2$ SYM [37–39], the generating function $tr\,\exp(\lambda\phi)$ of the chiral correlators $\langle tr\,\phi^J\rangle$ can be represented as

$$\langle tr\, e^{\lambda\phi}\rangle = \langle tr\, e^{\lambda\phi}\rangle_{cl} + \frac{1}{Z}\sum_K q^K \int d^k\chi \sum_i \prod_\ell (1-T_\ell^\lambda)\, e^{\lambda\chi_i}\,\mathcal{Z}_K(\chi)\,, \tag{6.1}$$

where the factors $(1-T_\ell^\lambda)$ with $T_\ell \equiv e^{\epsilon_\ell}$ properly take care of the volume factor.[4] $\mathcal{Z}_K$ is the integrand in the instanton partition functions (4.3) and

$$Z = \sum_K q^K \int d^k\chi\, \mathcal{Z}_K(\chi) \tag{6.2}$$

is the partition function. Thus to compute a specific correlator $\langle tr\,\phi^J\rangle$ in the contour integral one makes an insertion

$$\begin{aligned} O_{J,K}(\{\chi_I\}) = \sum_{I=1}^{K}\Bigg[\chi_I^J - \sum_{i=1}^{4}(\chi_I+\epsilon_i)^J + \sum_{i<j}^{4}(\chi_I+\epsilon_i+\epsilon_j)^J \\ - \sum_{i<j<k}^{4}(\chi_I+\epsilon_i+\epsilon_j+\epsilon_k)^J + (\chi_I+\epsilon_1+\epsilon_2+\epsilon_3+\epsilon_4)^J\Bigg] \end{aligned} \tag{6.3}$$

so that

$$\langle tr\,\phi^J\rangle = \frac{1}{Z}\sum_K q^K \int d^k\chi\, \mathcal{Z}_K(\chi)\, O_{J,K}(\{\chi\}) \tag{6.4}$$

Remarkably, the normalized correlators are, unlike the partition function Z itself, well defined even at the limit when ϵ's vanish. Direct calculations up to q^3 are not difficult:

$$\begin{aligned} \langle tr\,\phi^2\rangle &= -2\sum_{u=1}^{n} a_u^2 \\ \langle tr\,\phi^4\rangle &= 2\sum_{u=1}^{n} a_u^4 + 192\sqrt{A_n}\; q - 96\, A_{n-2}\, q^2 + 768\sqrt{A_n}\, A_{n-4}\, q^3 + \cdots \\ \langle tr\,\phi^6\rangle &= -2\sum_{u=1}^{n} a_u^6 + 1440\, A_{n-1}\, q^2 - 7680\,\sqrt{A_n}\, A_{n-3}\, q^3 + \cdots \\ \langle tr\,\phi^8\rangle &= 2\sum_{u=1}^{n} a_u^8 - 6720\, A_n\, q^2 + 35840\,\sqrt{A_n}\, A_{n-2}\, q^3 + \cdots \end{aligned} \tag{6.5}$$

where the first term gives the classical contribution to the correlator. To avoid lengthy expressions we did not present the gravitational corrections here but they can be obtained in a similar way.

[4] The domain of integration of (6.1) is the entire moduli space. The latter includes also the space-time translational zero modes whose contribution is cancelled by $\prod_\ell (1-T_\ell)$.

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The first non trivial correlator $\langle tr\,\phi^4\rangle$ in the list can be related to the derivative of the prepotential. This can be seen using the identity

$$O_{4,K}(\{\chi_I\}) = 24K\,\epsilon_1\epsilon_2\epsilon_3\epsilon_4 \tag{6.6}$$

that implies

$$\langle tr\,\phi^4\rangle = 2\sum_{u=1}^{n} a_u^4 + 24\,q\partial_q F = tr\,\phi_{\rm cl}^4 + 24\sum_K K F_K q^K \tag{6.7}$$

Alternatively this can be seen by noticing that each $\phi \sim F_{\mu\nu}\theta\gamma^{\mu\nu}$ in $tr\,\phi^4$ soaks precisely two out of the eight fermionic zero modes in the instanton background and therefore all together the amplitude in the K-instanton sector is given by $\int t_8 tr\,F^4 \sim K$ time the normalized centered partition function F_K [53]. The remaining correlators $tr\,\phi^J$ with $J > 4$ give new informations about the chiral ring of the theory beyond the prepotential.

In a similar way we can compute chiral correlators in the case of $\mathbb{R}^6 \times \mathbb{R}^4/\mathbb{Z}_2$. Now the relevant insertion is

$$O_J(\chi_I, K) = \sum_{I=1}^{K}\left[\chi_I^J - (\chi_I+\epsilon_1)^J - (\chi_I+\epsilon_2)^J + (\chi_I+\epsilon_1+\epsilon_2)^J\right] \tag{6.8}$$

and one finds

$$\begin{aligned}
\langle tr\,\phi^2\rangle =&\ -\sum_{u=1}^{n} 2a_u^2 - 4\sqrt{A_n}\,q - 2\,A_{n-1}\,q^2 - 4\sqrt{A_n}\,A_{n-2}\,q^3\\
&-2(A_{n-2}A_{n-1} + 5A_nA_{n-3})q^4\\
&-4\sqrt{A_n}\,(A_{n-2}^2 + 3A_{n-1}A_{n-3})\,q^5 + \cdots\\
\langle tr\,\phi^4\rangle =&\ \sum_{u=1}^{n} 2a_u^4 - 12A_n\,q^2 - 16\sqrt{A_n}\,A_{n-1}\,q^3\\
&-6(A_{n-1}^2 + 6A_nA_{n-2})q^4 - 48(A_{n-1}A_{n-2} + A_nA_{n-3})q^5 + \cdots\\
\langle tr\,\phi^6\rangle =&\ -\sum_{u=1}^{n} 2a_u^6 - 40\sqrt{A_n}\,A_n\,q^3 - 90A_nA_{n-1}q^4\\
&-72\sqrt{A_n}\,(A_{n-1}^2 + 3A_nA_{n-2}\,)q^5 + \cdots\\
\langle tr\,\phi^8\rangle =&\ \sum_{u=1}^{n} 2a_u^8 - 140A_n^2q^4 - 448\sqrt{A_n}\,A_n\,A_{n-1}q^5 + \cdots
\end{aligned} \tag{6.9}$$

In this case the Matone relation takes the form

$$\langle tr\,\phi^2\rangle = -2\sum_{u=1}^{n} a_u^2 - 2q\partial_q F \tag{6.10}$$

Note also that calculations up to q^7 strongly suggest that in the conformal case SO(4) the exact expression for the $\langle tr\,\phi^2\rangle$ is

$$\langle tr\,\phi^2\rangle = -2\left(a_1^2 + a_2^2 - \frac{1}{4}\,q^2(\epsilon_1^2 + \epsilon_1\epsilon_2 + \epsilon_2^2)\right)\frac{1}{1-q^2} - 4a_1a_2\,\frac{q}{1-q^2}\,. \tag{6.11}$$

This result is easily derived from (5.5) after taking the derivative with respect to q, according to (6.10).

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A SO(2n) prepotential

Here we give the expression for (4.6) up to 5 instantons (i.e. up to q^5) and up to the 4th order in the gravitational corrections

$$
\begin{aligned}
F(a_u,\epsilon_\ell) = {}&\\
&8\sqrt{A_n}q + q^2\left(-2A_{n-2} + \frac{1}{4}A_{n-3}g_2 - \frac{1}{64}A_{n-4}(g_2^2+2g_4)\right)\\
&+8\sqrt{A_n}q^3\left(\frac{4}{3}A_{n-4} - \frac{5}{6}A_{n-5}g_2 + \frac{1}{96}A_{n-6}(25g_2^2+34g_4)\right)\\
&+q^4\Bigg[-\frac{1}{2}A_{n-3}^2 - A_{n-2}A_{n-4} - 17A_{n-1}A_{n-5} - 113A_nA_{n-6}\\
&\qquad+\frac{1}{8}(3A_{n-3}A_{n-4}+19A_{n-2}A_{n-5}+195A_{n-1}A_{n-6}+1491A_nA_{n-7})g_2\\
&\qquad-\frac{3}{128}A_{n-4}^2(g_2^2+2g_4) - \frac{1}{64}A_{n-3}A_{n-5}(15g_2^2+14g_4)\\
&\qquad-\frac{3}{64}A_{n-2}A_{n-6}(41g_2^2+50g_4) - \frac{3}{64}A_{n-1}A_{n-7}(389g_2^2+442g_4)\\
&\qquad-\frac{1}{64}A_nA_{n-8}(9515g_2^2+11414g_4)\Bigg]\\
&+8\sqrt{A_n}q^5\Bigg[\frac{4}{5}\left(\frac{3}{2}A_{n-4}^2+7A_{n-3}A_{n-5}+23A_{n-2}A_{n-6}+87A_{n-1}A_{n-7}+263A_nA_{n-8}\right)\\
&+\frac{1}{2}(-9A_{n-4}A_{n-5}-29A_{n-3}A_{n-6}-109A_{n-2}A_{n-7}-413A_{n-1}A_{n-8}-1389A_nA_{n-9})g_2\\
&+\frac{1}{160}A_{n-5}^2(205g_2^2+218g_4) + \frac{3}{80}A_{n-4}A_{n-6}(145g_2^2+154g_4)\\
&+\frac{1}{80}A_{n-3}A_{n-7}(1625g_2^2+1738g_4) + \frac{1}{80}A_{n-2}A_{n-8}(6325g_2^2+6802g_4) \qquad \text{(A.1)}\\
&+\frac{1}{80}A_{n-1}A_{n-9}(24265g_2^2+26378g_4) + \frac{1}{80}A_nA_{n-10}(86645g_2^2+97522g_4)\Bigg] + \cdots \qquad \text{(A.2)}
\end{aligned}
$$

B SO(8) case

The case $n = 4$ is special in the sense that q is dimensionless and the theory is conformal. Putting $n = 4$ in (A.2) and promoting the expectation values to the respective fields,

$$\begin{aligned} tr\,\phi^2 &= -2\sum_{u=1}^{4} a_u^2\,; \quad tr\,\phi^4 = 2\sum_{u=1}^{4} a_u^4\,; \\ tr\,G^2 &= -2\sum_{\ell=1}^{4} \epsilon_\ell^2\,; \quad tr\,G^4 = 2\sum_{\ell=1}^{4} \epsilon_\ell^4 \end{aligned} \tag{B.1}$$

one finds

$$\begin{aligned} F(\phi,G) &= 8\mathrm{Pf}\phi\left(q+\frac{4}{3}q^3+\frac{6}{5}q^5+\cdots\right) \\ &\quad +\frac{1}{2}tr\,\phi^4\left(q^2+\frac{1}{2}q^4+\cdots\right)-\frac{1}{4}\left(tr\,\phi^2\right)^2\left(q^2+q^4+\cdots\right) \\ &\quad +\left(\frac{1}{16}tr\,\phi^2\;tr\,G^2-\frac{1}{64}tr\,G^4-\frac{1}{256}(tr\,G^2)^2\right)\left(q^2+\frac{3}{2}q^4+\cdots\right) \end{aligned}$$

This suggests that the exact expression in all orders of q would be

$$\begin{aligned} F(\phi,G) = \sum_{k=0}^{\infty}\Bigg\{ & 8\mathrm{Pf}\phi\sum_{l|2k+1}\frac{1}{l}\,q^{2k+1} \\ & +\frac{1}{2}tr\phi^4\sum_{l|k}\frac{1}{l}\left(q^{2k}-q^{4k}\right)-\frac{1}{4}\left(tr\,\phi^2\right)^2\sum_{l|k}\frac{1}{l}\left(q^{2k}-\frac{1}{2}q^{4k}\right) \\ & +\left(\frac{1}{16}tr\,\phi^2\;tr\,G^2-\frac{1}{64}tr\,G^4-\frac{1}{256}\left(tr\,G^2\right)^2\right)\sum_{l|k}\frac{1}{l}\,q^{2k}\Bigg\} \end{aligned} \tag{B.2}$$

in agreement with the heterotic results [40–47] (up to normalizations of fields and traces)

$$\begin{aligned} \Delta_{F^4} &= -\log\frac{|\eta(T)|^4}{|\eta(\frac{T}{2})|^4}+\ldots \\ \Delta_{(F^2)^2} &= -\frac{1}{2}\log\frac{T_2U_2|\eta(T/2)|^8|\eta(U)|^4}{|\eta(T)|^4}+\ldots \\ \Delta_{R^4} &= 4\Delta_{(R^2)^2} = 2\Delta_{R^2F^2} = -\frac{1}{16}\log T_2U_2|\eta(T/2)|^4|\eta(U)|^8+\ldots \end{aligned} \tag{B.3}$$

where dots refer to moduli independent contributions.

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